Pedestrian Loads with Statistical Frequency Distribution

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I. Introduction

In recent years pedestrian loads have received increased attention, largely due to vibration problems for light pedestrian bridges. In many cases light pedestrian bridges need some kind of additional damping, and this raises the question of reliable, yet simple, methods for assessing the expected vibration level of the bridge in its original undamped configuration, and also if needed the necessary level of additional damping. The traditional paradigm, see e.g. Rainer et al. [1], is based on an estimated design value of the pedestrian load that could cause resonance - typically in the range 1.8–2.0 Hz. The pedestrian load was assumed to act at the resonance frequency, giving the response amplitude

$$x_1 \simeq \frac{1}{2\zeta_{\text{eff}}} \frac{F}{k_1} \tag{1}$$

where ζ_{eff} is the effective damping ratio, including the effect of damping devices. This leads to a fairly conservative design, if using the full load intensity. Alternatively, a simple white noise approximation of the load gives the response variance

$$\sigma_1 = \frac{1}{\sqrt{2\zeta_{\text{eff}}}} \frac{\sqrt{\pi\omega_1 S_F}}{k_1} \tag{2}$$

where S_F is the two-sided spectral density of the load at the resonance frequency. While the single-frequency response is inversely proportional to ζ_{eff}, the wide-band response is inversely proportional to $\zeta_{\text{eff}}^{-1/2}$. A more realistic response variance can be obtained by accounting for the frequency distribution of the pedestrian footfall frequency.

II. Pedestrian Load Model

Measured walking load characteristics, see e.g. [1], [2], [3] indicate a coefficient of variation of around 0.7. This is represented by a normal distribution for the individual step frequency,

$$\frac{S_F^*(\omega)}{\sigma_F^{*2}} = \frac{1}{\sqrt{2\pi}\sigma_\omega} \exp\left\{ -\frac{1}{2}\left(\frac{\omega-\omega_p}{\sigma_\omega}\right)^2\right\} \tag{3}$$

This distribution is replaced by a rational 'equivalent',

$$\frac{S_F'(\omega)}{\sigma_F^2} = \frac{1}{\pi} \frac{\beta}{(\omega_p-\omega)^2+\beta^2} \tag{4}$$

with the same peak frequency ω_p and the bandwidth parameter β, illustrated in Fig. 1. Matching peak value and peak curvature gives

$$\beta = \sqrt{2}\,\sigma_\omega\,, \qquad \sigma_F^2 = \sqrt{\pi}\,\sigma_F^{*2} \tag{5}$$

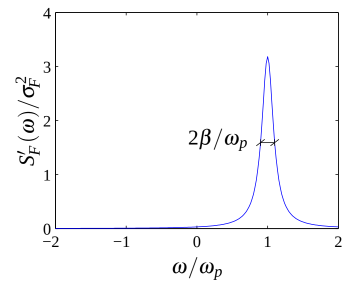

Fig. 1. One-sided spectral density $S_F'(\omega)$

and the measured coefficient of variation thus corresponds to $\beta/\omega_p \simeq 0.1$. This bandwidth is considerably larger that that of the bridge response function, thereby permitting analytical estimates as well as a detailed calculation of the response variance.

A simple change of variables leads to the two-sided spectral load spectral density

$$\frac{S_F(\omega)}{\sigma_F^2} = \frac{\zeta_0\omega_0}{\pi} \frac{\omega_0^2+\omega^2}{(\omega_0^2-\omega^2)^2+4\zeta_0^2\omega_0^2\omega^2} \tag{6}$$

with $\omega_0 = \omega_p$ and $\zeta_0 \simeq \beta/\omega_p$. This spectral load density enables an 'exact' response analysis in terms of the Lyapunov equation, providing the variances of the response, and the possibility of including damping devices in the structural system.

The spectral density of the load also permits a simple approximate response analysis of the undamped structure. When using that the bandwidth of the load ζ_0 is much larger then the spectral width of the structural transfer spectrum ζ_1, the structural response variance for load around resonance is

$$\sigma_1^2 = \frac{\pi}{2} \frac{1}{\omega_1^3\zeta_1} \left(\frac{\omega_1^2}{k_1}\right)^2 S_F(\omega_0) \tag{7}$$

When inserting the load spectral density from (6), the response variance is found as

$$\sigma_1 \simeq \frac{1}{2\sqrt{\zeta_1\zeta_0}} \frac{\sigma_F}{k_1} \tag{8}$$

This format combines harmonic and spectral formats (1) and (2), and can be used to estimate the need for damping devices.

References

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